

## Differences between oral and written calculation: evidence from cognitive neuropsychology from six brain-damaged patients

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**Título:** Diferencias entre cálculo oral y escrito: evidencias desde la neuropsicología cognitiva a partir de seis casos de daño cerebral.

**Resumen:** Introducción: El estudio de pacientes con daño cerebral adquirido pone de manifiesto la existencia de varias dobles disociaciones en el sistema de cálculo. En este trabajo nos centramos en la doble disociación observada entre cálculo oral y cálculo escrito. Método: Instrumento: Batería de Evaluación del Procesamiento Numérico y Cálculo. Participantes: Seis pacientes con daño cerebral adquirido que presentan distintas alteraciones en el procesamiento de los números y el cálculo. Análisis de los datos: Diferencia de proporciones. Resultados: MC y BET presentan alterado el cálculo escrito pero conservan el cálculo oral (suma, resta y multiplicación). Lo mismo sucede a MNL en suma y multiplicación y a PP en la resta. El patrón inverso, se observa en IRS y ACH con alteraciones en cálculo escrito y preservación del oral (en multiplicación y resta, respectivamente). Conclusiones: Los resultados ponen de manifiesto la independencia funcional entre cálculo oral y escrito. Esto podría indicar que el sistema de cálculo no es un todo unitario responsable de cualquier tarea de cálculo, sino que posiblemente sea un sistema multicomponential en el que intervendrían distintos procesos y de diferente naturaleza.

**Palabras clave:** Neuropsicología cognitiva; cálculo; daño cerebral; procesamiento numérico.

**Abstract:** Introduction: The study of patients with acquired brain injury shows the existence of several double dissociations in the calculation system. In this paper, we focus on the double dissociation observed between oral and written calculation. Method: Instrument: Battery of Evaluation and Numerical Processing and Calculation. Participants: Six patients with acquired brain injury who have different alterations in the processing of numbers and calculations. Data analysis: Difference of proportions. Results: MC and BET have impaired the written calculation but they preserve oral calculation (addition, subtraction and multiplication). The same is observed in MNL for addition and multiplication and in PP for subtraction. The reverse pattern is observed in IRS and ACH who have alterations in written calculation but preserve oral calculation (in multiplication and subtraction, respectively). Conclusions: The results demonstrate the functional independence of oral and written calculation. This could indicate that the calculation system is not unitary and responsible for any calculation task, but a multi-componential system involving different processes and of a different nature.

**Key words:** Cognitive neuropsychology; calculation; brain injury; number processing.

### Introduction

Processing numeric and calculation involve a set of heterogeneous processes (Alameda & Cuetos, 1997; Alameda, Cuetos, & Brysbaert, 2003; Salguero & Alameda, 2003, 2010, 2013; Salguero, Lorca, & Alameda, 2003, 2004) and there is increasing evidence of this fact provided by cognitive neuropsychology because after brain damage, some skills may be altered while others remain intact. For example, single-case studies show that number processing (e.g., comprehension and/or production arabic and/or verbal numerals) is independent of the mechanisms in charge of calculation (e.g., Alameda, Salguero, & Lorca, 2007; Borges, García-Solís, & Borrego, 1999; Cipolotti & Butterworth, 1995; Dehaene & Cohen, 1997; García-Orza, León-Carrión & Vega, 2003; Warrington, 1982).

Concerning calculation itself, the study of brain-damaged patients shows that it could be made up of different components that can function independently. Specifically, regarding oral and written calculation, empirical evidence reveals that there is a double dissociation. On the one hand, the case of HAR (McNeil & Warrington, 1994), who presents a selective deficit for addition and multiplication of arabic written numbers, but preserves subtraction of written numbers and oral calculation. However, the dissociation between oral and written calculation is also present in two patients who suffer

pure alexia studied by Cohen and Dehaene (1995): both patients correctly carry out simple additions, presented orally, but they commit between 62.5 and 70% of errors when the same operations are presented in arabic form.

This evidence, revealed through studies of brain-damaged patients, should be interpreted and explained by the different models of numeric processing and calculation, although not all these models refer to calculation processes. Mainly, the models of McCloskey et al. (McCloskey, Caramazza, & Basili, 1985; McCloskey, Sokol, & Goodman, 1986) and that of Dehaene and Cohen (1995, 1997) are the ones attempting to explain arithmetic processing and calculation in addition to numeric processing.

The model of McCloskey et al. (1985, 1986) proposes number processing as a system made up of different modules operating autonomously, and each one is specialized in a certain function (arabic and verbal comprehension and production) with the peculiarity that this model postulates obligatory access to the magnitude represented by the number for any recoding or calculation task. Therefore, the model does not contemplate any type of asemantic number processing. The semantic representation of quantity represented by the number would, in turn, be directly connected to the calculation system, which would be made up of three components, also independent of each other: processing arithmetic signs, numerical data, and calculation procedures (Figure 1).

This model predicts that, in order to carry out an oral calculation task, in addition to the calculation system and the internal abstract quantity representation, the input system of verbal numeric stimuli and the verbal production system

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must be preserved. And to carry out a task of written calculation, as with oral calculation, in addition to preserving the calculation system and the abstract quantity representation,

the model predicts that both the arabic stimulus input system and the arabic production system must be preserved.

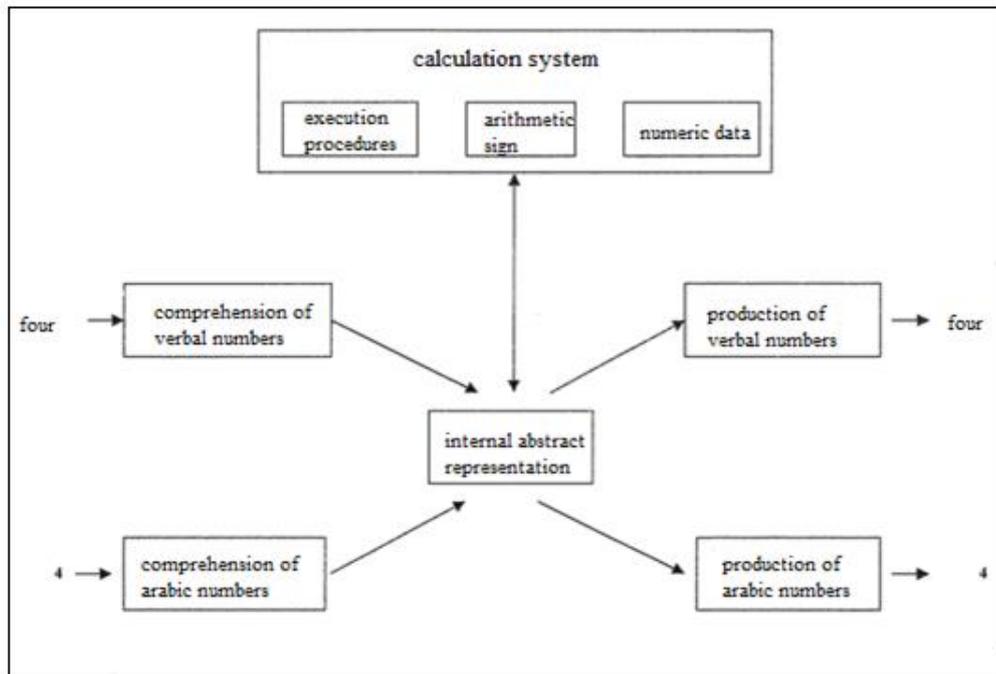


Figure 1. Model of McCloskey et al. (1985).

The anatomical functional model Dehaene and Cohen (1995, 1997) postulates three types of mental representations for numbers, two asemantic types (visual-arabic form and verbal word structure) and one semantic type (analogical magnitude representation). In the visual-arabic form, numbers are represented as chains of digits; that is, a visuo-spatial representation. It is located in the inferior ventral occipitotemporal areas of both hemispheres. The verbal word structure is the representation of numbers in the form of sequences of syntactically organized words, and it is located in the classic perisylvian language areas of the left hemisphere. Lastly, the meaning of numbers (quantitative and lexical) is portrayed in the analogical magnitude representation, and in this area, the quantity or magnitude associated with a number is retrieved and can thus be related to other quantities. This representation is located in the inferior parietal of both hemispheres and is responsible for magnitude comparison, as well as for calculation operations requiring semantic elaboration, such as subtraction. In contrast, retrieval of arithmetic data from tables, such as multiplication and sometimes addition, depends directly on the verbal representation of the number word and, therefore, these data can be accessed without semantic mediation (Figure 2).

According to this model, the visual-arabic form of the number, the representation of the verbal word structure and—depending on the arithmetical operation—the analogical magnitude representation would all intervene in performance of written calculation. Semantic elaboration would be

necessary for subtraction but not for multiplication, and sometimes not for addition either because numeric data are stored as verbal routines.

Concerning oral calculation, this model predicts that oral addition and multiplication could be solved exclusively with the representation of the verbal word structure. Regarding oral subtraction, in addition to the verbal word structure, the analogical magnitude representation would be necessary for the processes of semantic elaboration.

The diverse models of number processing and calculation indicate that the access pathways for one type of calculation may differ from those of another, and that the mechanisms involved in oral and/or written calculation can also vary depending on the arithmetic operation. Moreover, language can have an impact on the performance of calculation tasks precisely because of the verbal structure of the name of the numbers (e.g., Ellis & Hennelly, 1980; Hoosain & Salili, 1988), but there are no works in Spanish in this regard, and works in other languages are scarce.

On the basis of these antecedents, in this work, we study oral and written calculation in patients with acquired brain damage, in order to determine which processes are impaired in each patient and how their performances can be interpreted in the context of the current models.

## Method

### Participants

In this work, a total of six patients participated. They all had acquired brain damage. Table 1 shows the main characteristics of the medical diagnosis and the neuropsychological assessment of each patient.

### Instrument

The “Batería para la Evaluación del Procesamiento Numérico y el Cálculo” [Battery for the Assessment of Numeric Processing and Calculation] of Salguero and Alameda (2011) was applied. The written calculation test is made up of 36 additions, 30 subtractions, and 27 multiplications, in one half of which a number must be "carried" but not in the other half, and all operations are presented vertically. The oral calculation test consists of 15 additions, 15 subtractions, and 10 multiplications, of 1 to 2 digits and, again, one half of the operations require a number to be "carried".

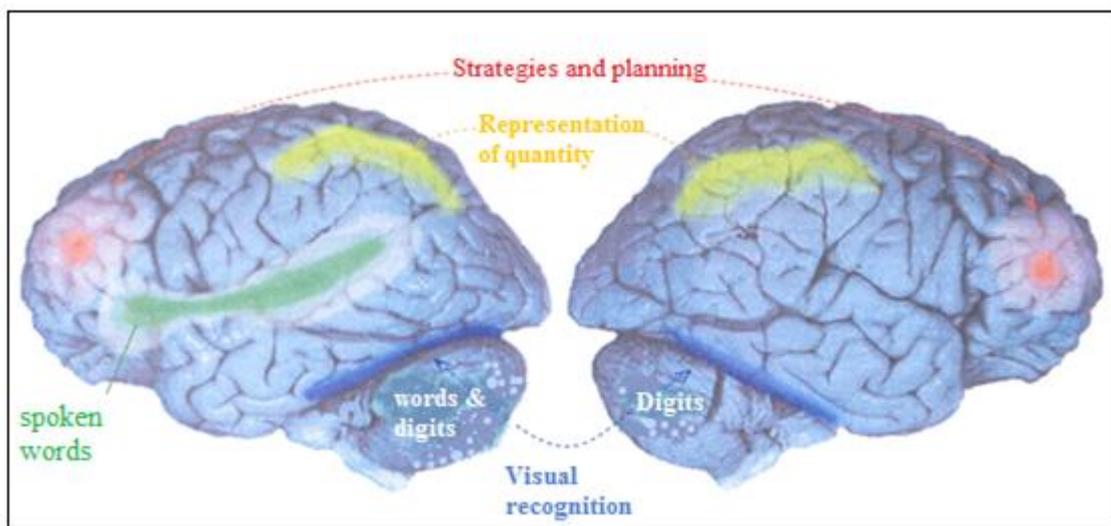


Figure 2. Anatomical functional model of number processing (Dehaene & Cohen, 1995, 1997).

### Data Analysis

The statistical analyses of data were performed with a difference of proportions (Moore & McCabe, 2001; Pryce,

2005). This procedure allows comparing the performance of a patient with regard to a control group, even in situations where a different total number of items may have been used in the two cases.

Table 1. Summary of the patients' main characteristics.

	ACH	BET	ISR	MC	MNL	PP
Year of birth	1966	1963	1989	1966	1956	1950
Sex	Male	Female	Male	Female	Male	Male
Main medical diagnosis	Traumatic diffuse axonal injury	Trauma in frontal lobes	Traumatic diffuse axonal injury	Trauma in right temporal and left basal ganglia	Infarction of the left middle cerebral artery	Infarction in left parietal lobe
	Mild deficit	Deficit in sustained attention	Mild deficit	Not assessed	Mild deficit	Mild deficit
	Attention					
	Correct	Moderate anomaly	Moderate deficit in fluency	Type Broca motor aphasia	Correct	Mixed aphasia
	Language					
	Mild deficit in working memory	Implicit learning deficit	Moderate deficit in short- and long-term verbal and visual	Moderate deficit in episodic	Moderate deficit in working memory	Mild deficit in verbal and visual
	Moderate deficit in verbal memory			Moderate deficit in short-term verbal and visual		
	Memory					
	Impaired	Not assessed	Correct	Not assessed	Impaired	Cognitive functions impaired
	Executive functions					

## Results

Table 2 presents the results of each patient in the Numerical Recoding and Calculation blocks of the Battery for the Assessment of Number Processing and Calculation (Salguero & Alameda, 2011).

The patients assessed present impairments in written arithmetic operations while preserving the oral form, and vice versa. The double dissociations are observed in the diverse arithmetic operations: addition, subtraction, and multiplication.

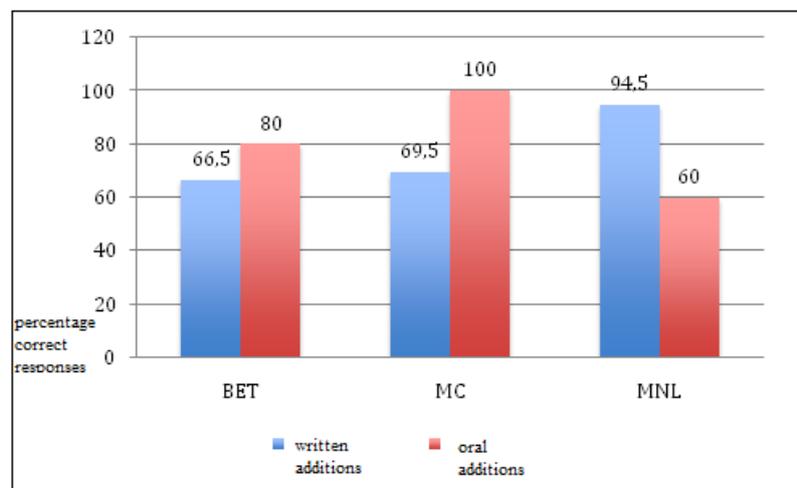
**Table 2.** Patients' Results in the Numerical Recoding and Calculation Blocks.

Blocks and tests	ACH		BET		ISR		MC		MNL		PP		
	% Correct responses	<i>p</i>	% Correct responses	<i>p</i>	% Correct responses	<i>p</i>	% Correct responses	<i>p</i>	% Correct responses	<i>p</i>	% Correct responses	<i>p</i>	
Numerical Recoding	Identification	100	1	100	1	100	1	100	1	100	1	95	.23
	Repetition	100	1	100	1	100	1	*		91	.01	88	.00
	Reading arabic numbers	100	1	100	1	100	1	*		97	.15	*	
	Reading verbal numbers	87.5	.01	100	.57	100	.57	*		100	.57	*	
	Arabic-verbal recoding	100	.57	100	.57	100	.57	94	.17	80	.00	96	.45
	Verbal-arabic recoding	78	.00	94.5	.31	100	.57	94.8	.64	100	.57	100	.57
	Verbal dictation	100	1	100	1	100	1	*		100	1	*	
	Arabic dictation	87.5	.1	100	1	100	1	*		100	1	65	.00
Calculation	Verification	78	.07	81.5	.23	100	1	100	1	89	.23	78	.07
	Numerical reasoning	63.5	.00	76	.02	100	.31	83.5	.08	50	.00	100	.32
	Written addition	94.5	.55	66.5	.00	97.5	1	69.5	.00	94.5	.55	69	.00
	Written subtraction	66.5	.00	54	.00	97	.66	60.5	.00	73.5	.01	96	.66
	Written multiplication	53	.00	33	.00	69	.00	48.5	.00	93	.86	66	.02
	Oral addition	66.5	.06	80	.28	100	.24	100	.3	60	.03	60	.03
	Oral subtraction	73.5	.14	93.5	1	93.5	1	84.5	0.45	60	.03	46.5	.00
	Oral multiplication	40	.00	70	.26	70	.26	90	1	50	.05	3	.00

(\*) Could not be assessed

With regard to addition, the results are represented in Figure 3. As can be observed, on the one hand are the cases of patients BET and MC, who present impairment in written addition (66.5% correct responses,  $p < .00$ ; 69.5% correct responses,  $p < .00$ , respectively) but they preserve oral additions (80% correct responses,  $p = .28$ ; 100% correct re-

sponses,  $p = .3$ , respectively). The double dissociation is completed with the case of MNL, who retains written addition (94.5% correct responses,  $p = .55$ ) but who presents impairment in oral addition (60% correct responses,  $p = .03$ ).



**Figure 3.** Results of Patients BET, MC, and MNL in oral and written additions.

A double dissociation was also observed in subtractions (Figure 4). On the one hand, patients ACH, BET, and MC present impairment in written subtraction (66% correct re-

sponses,  $p < .00$ ; 54% correct responses,  $p < .00$ ; 60.5% correct responses,  $p < .00$ , respectively) but they retain oral subtraction (73.5% correct responses,  $p = .14$ ; 93.5% correct

responses,  $p = 1$ ; 84.5% correct responses,  $p = .45$ , respectively). In contrast, patient PP presents the inverse pattern, that is, impaired oral subtraction (46.5% correct responses,  $p < .00$ ) but retaining written subtraction (96% correct responses,  $p = .66$ ).

The results are represented in Figure 5. Patients BET, ISR, and MC present impairment in written multiplication (33% correct responses,  $p < .00$ ; 69% correct responses,  $p <$

.00; 48.5% correct responses,  $p < .00$ , respectively) but they preserve oral multiplication (70% correct responses,  $p = .26$ ; 70% correct responses,  $p = .26$ ; 90% correct responses  $p = 1$ , respectively). Patient MNL presents the inverse performance pattern, that is he retains written multiplication (93% correct responses,  $p = .86$ ) but presents impairment for oral multiplication (50% correct responses,  $p = .05$ ) so the double dissociation is complete.

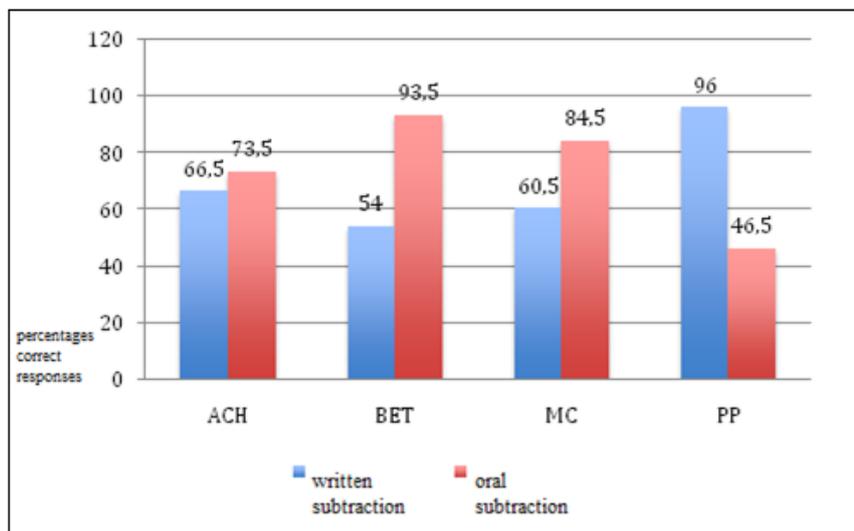


Figure 4. Results of Patients ACH, BET, MC, and PP in oral and written subtractions.

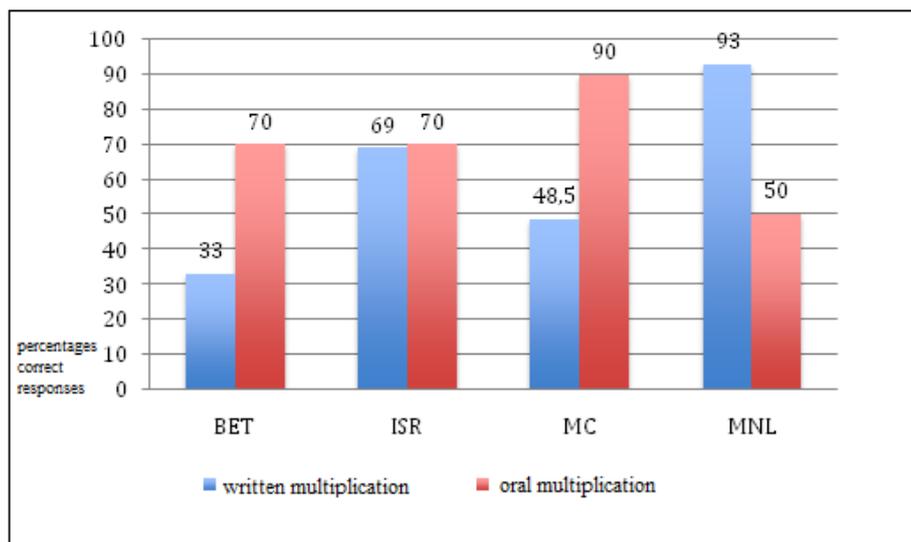


Figure 5. Results of Patients BET, ISR, MC, and MNL in oral and written multiplications.

## Discussion and Conclusions

Our results reveal the existence of a double dissociation between oral and written calculation, which is also displayed in addition, subtraction, and multiplication.

The model of McCloskey et al. (1985) postulates that the input system of arabic stimuli is independent of the input system of verbal numeric stimuli. Therefore, it could explain the cases in which written calculation is retained and oral calculation is impaired, alluding to the fact that the deficit is located in the verbal number input system, so that such in-

puts would not have access to the internal abstract representation and could not reach the calculation system. This means that, in other numeric tasks involving this verbal number input system, deficient performances would also be observed, for example, in the verbal-arabic recoding test.

This assumption is seen in two of our patients, MNL, who retains written additions and multiplications, and PP, who retains written subtraction. However, these patients do not present difficulties in the verbal-arabic recoding task (Table 2); therefore, the evidence does not match the assumptions of the model.

Regarding the inverse pattern, that is, when oral calculation is retained and written calculation is impaired, this model would explain it as a consequence of a selective impairment of the system of verbal number comprehension, with the comprehension of arabic numbers remaining intact, so that, applying the assumptions of the model, these patients would also present impairment in other tasks that require processing arabic inputs, for example, the arabic-verbal recoding task. Such would be the cases of BET and MC, who present impairments in written addition, subtraction, and multiplication, as well as the cases of IRS, with impaired written multiplication, and ACH, who presents impairment in written subtraction. However, these patients present no difficulties in arabic-verbal recoding (Table 2). Once again, the results of our patients are not consistent with the predictions of the model McCloskey et al. (1985).

However, our results are completely consistent with the anatomical functional model (Dehaene & Cohen, 1995, 1997). As seen before, according to this model, not all the arithmetic operations would be resolved in the same way, but instead this model proposes the existence of two pathways, depending on the operation. On the one hand, multiplications—and sometimes additions—would be resolved directly through an asemantic pathway, which in the case of written operations, would involve two types of representation, the visual-arabic form and the verbal word structure of the number; in the case of oral operations, only the verbal word structure would be necessary. On the other hand, regarding subtractions, the analogical magnitude representation would be necessary both for oral and written subtractions. In the case of written subtraction, this would also involve the visual-arabic form, whereas in oral subtraction, the verbal word structure of the number would intervene instead.

This model could explain the cases of BET and MC, who present impairments in written addition, subtraction, and multiplication but preserve these arithmetic operations in the oral modality. Therefore, according to the model, the impairment is located in the visual form of the number of the left hemisphere, which cannot transmit the information to the verbal system (representation of the word structure of the number) in order to retrieve the numeric data. The linguistic representation and the analogical magnitude repre-

sentation are both preserved; this would allow both patients to carry out arithmetic operations orally because the visual-arabic form does not intervene in this modality. However, performing other tasks that require processing arabic stimuli, for example, identifying arabic numbers, is intact in these patients (Table 2), and this is explained by the visual-arabic representation of the right hemisphere, which is preserved without any impairment.

However, this interpretation of the model does not explain the cases of ISR and ACH, because these patients present a deficit in written calculation that is only observed in one arithmetic operation, while all the other operations are preserved. Thereby, we cannot argue that ISR has an impairment in the visual-arabic representation of the left hemisphere that prevents him from performing written multiplications because this patient can perform written subtractions and additions, tasks that require the intervention of this representation. The same is observed in ACH, but with subtractions.

Therefore, the anatomical functional model (Dehaene & Cohen, 1995, 1997) can explain the cases in which the entire written calculation is impaired, but oral calculation is preserved, by referring to an impairment in the visual-arabic representation of numbers, at least of the left hemisphere, which is the one that connects with the linguistic representation (verbal word structure of the number) to retrieve the numeric datum. However, the model could not explain the cases in which not all arithmetic operations are impaired.

But it is more difficult for the anatomical functional model to explain the inverse performance pattern, that is, the cases like those of PP and MNL, who preserve written calculation but have impaired oral calculation.

These cases should be interpreted as a consequence of impairment in the verbal system itself, that is, in the verbal word structure of the number, but this is incompatible with the patients' results in written calculation because, according to the model, solving written calculations would also require linguistic representations to retrieve the numeric data. That is, according to this model, it is unfeasible for written calculation to be preserved if oral calculation is impaired because both of them require the intervention of the same representations and, in the case of written calculation, visual-arabic representation is also required.

Summing up, as a conclusion, we can state that our results confirm the existence of a double dissociation between oral and written calculation, indicating the functional independence of both types of calculation. That is, the cognitive mechanisms underlying both tasks seem to be different, so that one could propose that the calculation system is not a unitary whole that is responsible for any calculation task, but instead it may be a multi-component system in which different kinds of processes intervene, so that we frequently find brain-damaged patients with some skills impaired but who preserve other skills intact.

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